Tuning Fork Frequencies

Tuning forks are steel bars which acoustically resonate at a fixed pitch. They are frequently used to tune musical instruments due to their consistency and ease-of-use. The standard tuning fork sounds A (440 Hz), which is the standard concert pitch. In general, the following formula can be used to determine the frequency of a given tuning fork.

$$f = \frac{1.875^2}{2\pi l^2}\sqrt{\frac{a^2 E}{12\rho}}$$

where f is the frequency the fork vibrates (Hz);

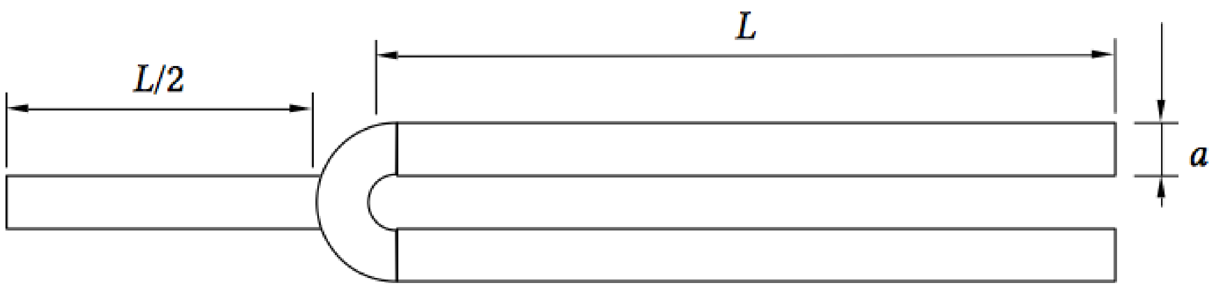
1.875 is the smallest positive solution of $\cos x \cosh x = -1$;

l is the length of the prongs (m) (typically 80–90 mm);

E is the Young's modulus of the material (Pa);

a is the edge of the square area A of the prong cross-section (m) (typically 4–5 mm); and

ρ is the density of the material ($kg{\cdot}m^{-3}$).



You will create a tuning fork model in two and three dimensions which adhere to the specifications given below, and then carry out several simulations. In each of these, you will determine the initial vibrational frequency of the model and the first nine overtones plus stresses. (Create a linear perturbation/frequency analysis step with ten modes using the "Lanczos" eigensolver.) I recommend using a = 4 mm and starting with L from 80–90 mm.

1. Calculate the characteristics of a tuning fork designed to sound 440 Hz as the dominant pitch. Then simulate this tuning fork as:

    1. A two-dimensional planar simulation using beam elements with thickness the same as the prong width (solid square cross-section of prong ■).

    2. A three-dimensional simulation using solid hexahedral elements (full integration). This may require partitioning the mesh as in the bottom-up meshing example. Do not use tetrahedra if at all possible.

    Report the modes of your default model, and then tune it such that the dominant mode falls within 5% of the correct tuning frequency (if necessary). Report the actual length of this calculated tuning fork.

2. Find dimensions for a middle C (261.26 Hz) tuning fork from the formula above. Construct and model this fork (in three dimensions).

   Report the modes of your default model, and then tune it such that the dominant mode falls within 5% of the correct tuning frequency (if necessary). Report the actual length of this calculated tuning fork.

Assume the tuning fork is made of aluminum, $E = 69 \times 10^9$ Pa, $\nu = 0.33$, and $\rho = 2.7 \times 10^{-3}$ g·mm^{-3}. You should verify whether this unit system is consistent and make certain that you carry out any unit transformations necessary before using the formula above.

You are responsible to show that your mesh converges reasonably and make a case for its sufficient accuracy. (A mesh density of 5% of the overall dimensions is usually a good starting point.) If the handle appears to be causing trouble, you can delete the handle and use a U-shaped geometry for the model; in any case, you need to constrain the base of the Y or U at the junction as it is a frequency node. I found that constraining the lines or points along its edge worked well. You should check the frequency range 20–30,000 Hz (a bit beyond the limits of human hearing).

(Intriguingly, tuning forks can be used to detect bone fractures noninvasively. A low C (128 Hz) tuning fork is toned on the suspected bone, causing the periosteum (bone membrane) to vibrate. This in turn causes nociceptors (pain receptors) to fire, resulting in a local sharp pain.)

You will document the simulations in a 8–10 page report (with figures) containing the sections:

- Problem description (tuning fork shape, grid, etc.)
- Numerical values (element parameters; number of nodes and their frequencies; boundary conditions; etc.)
- Computational times (CPU time to solve)
- Observations of numerical behavior (boundary conditions; mesh behavior, including convergence; etc.)
- Discussion of the physics (modes, etc.—is the difference between the formula and the actual value a constant ratio?)

Include the following plots in your report, with data from each case you will study:

- Mesh of model
- Contour plot of displacement in x and y directions
- Stills of animation of modal vibration; you may also wish to produce movie files

The report should be formatted with 1.5 line spacing, 1 inch margins on all sides, and set in 11 point serif font. Figures and tables should be numbered with labels and captions.