Ask the experts ANSYS CFD

MONTERREY

ANSYS®

CONVERGENCE

Introduction

Lecture Theme:

The accuracy of CFD results can be affected by different types of errors. By understanding the cause of each different error type, best practices can be developed to minimize them. Meshing plays a significant role in the effort to minimize errors.

Learning Aims:

You will learn:

- Four different types of errors
- Strategies for minimizing error
- Issues to consider during mesh creation such as quality and cell type
- Best practices for mesh creation

Learning Objectives:

You will understand the causes of error in the solution and how to build the mesh and perform the simulation in a manner that will minimize errors

Motivation for Quality

CFD-Results are used for many different stages of the design process:

- Design & optimization of components and machines
- Safety analyses
- Virtual prototypes

When undertaking a CFD model, consideration should be given to the purpose of the work:

- What will the results be used for?
- What level of accuracy will be needed?

Different Sources of Error

There are several different factors that combine to affect the overall solution accuracy. In order of magnitude:

- **Round-off errors**
  - Computer is working to a certain numerical precision
- **Iteration errors**
  - Difference between ‘converged’ solution and solution at iteration ‘n’
- **Solution errors**
  - Difference between converged solution on current grid and ‘exact’ solution of model equations
  - ‘Exact’ solution → Solution on infinitely fine grid
- **Model errors**
  - Difference between ‘exact’ solution of model equations and reality (data or analytic solution)

Round-Off Error

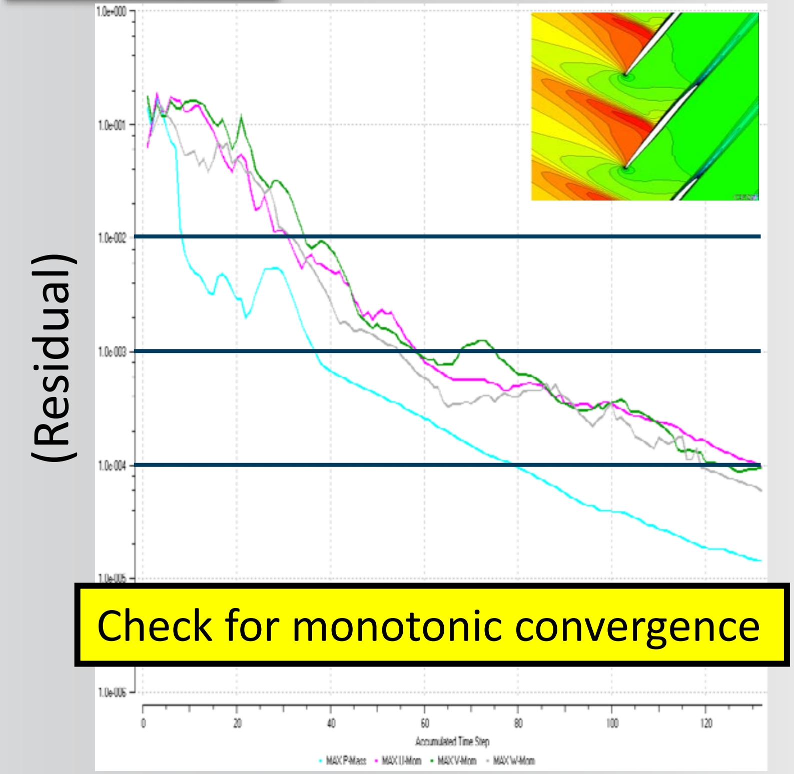
Inaccuracies caused by machine round-off:

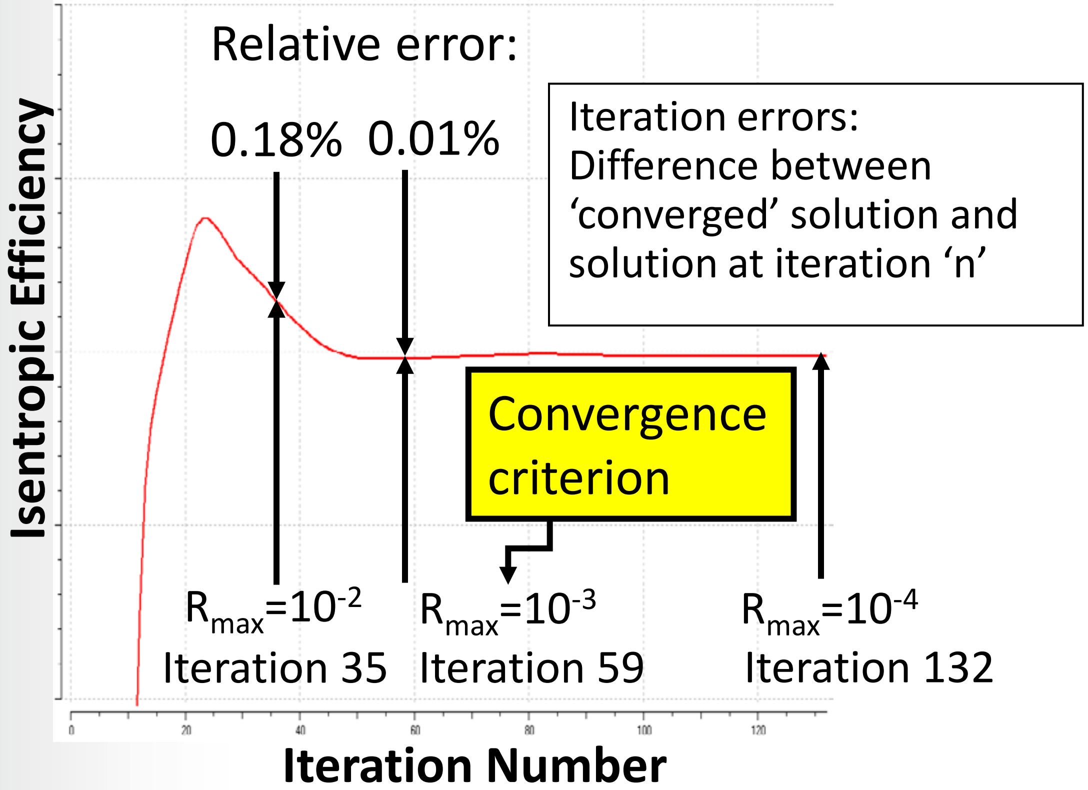
- High grid aspect ratios
- Large differences in length scales
- Large variable range

Procedure:

- Check above criteria
- Define target variables
- Calculate with:
  - Single-precision
  - Double-precision
- Compare target variables

Iteration Error Example: 2D Compressor Cascade





Iteration Error - Best Practice

- **Define target variables:**
  - Head rise
  - Efficiency
  - Mass flow rate
  - …
- **Select convergence criterion (e.g. residual norm)**
- **Plot target variables as a function of convergence criterion**
- **Set convergence criterion such that value of target variable becomes “independent” of convergence criterion**
- **Check for monotonic convergence**
- **Check convergence of global balances**

Discretization Error

All discrete methods have solution errors:

- Finite volume methods
- Finite element methods
- Finite difference methods
- ...

Difference between solution on a given grid and "exact' solution on an infinitely fine grid

$$e_h = f_h - f_{ex}$$

Exact solution not available → Discretization error estimation

Introduction | Error Types | Best Practices for Meshing | Summary

Discretization Error Estimation

Impinging jet flow with heat transfer

2-D, axisymmetric

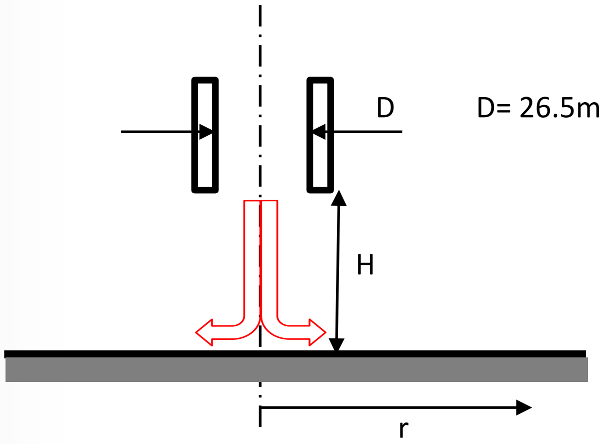
Compared Grids:

- 50 × 50 → 800 × 800

SST turbulence model

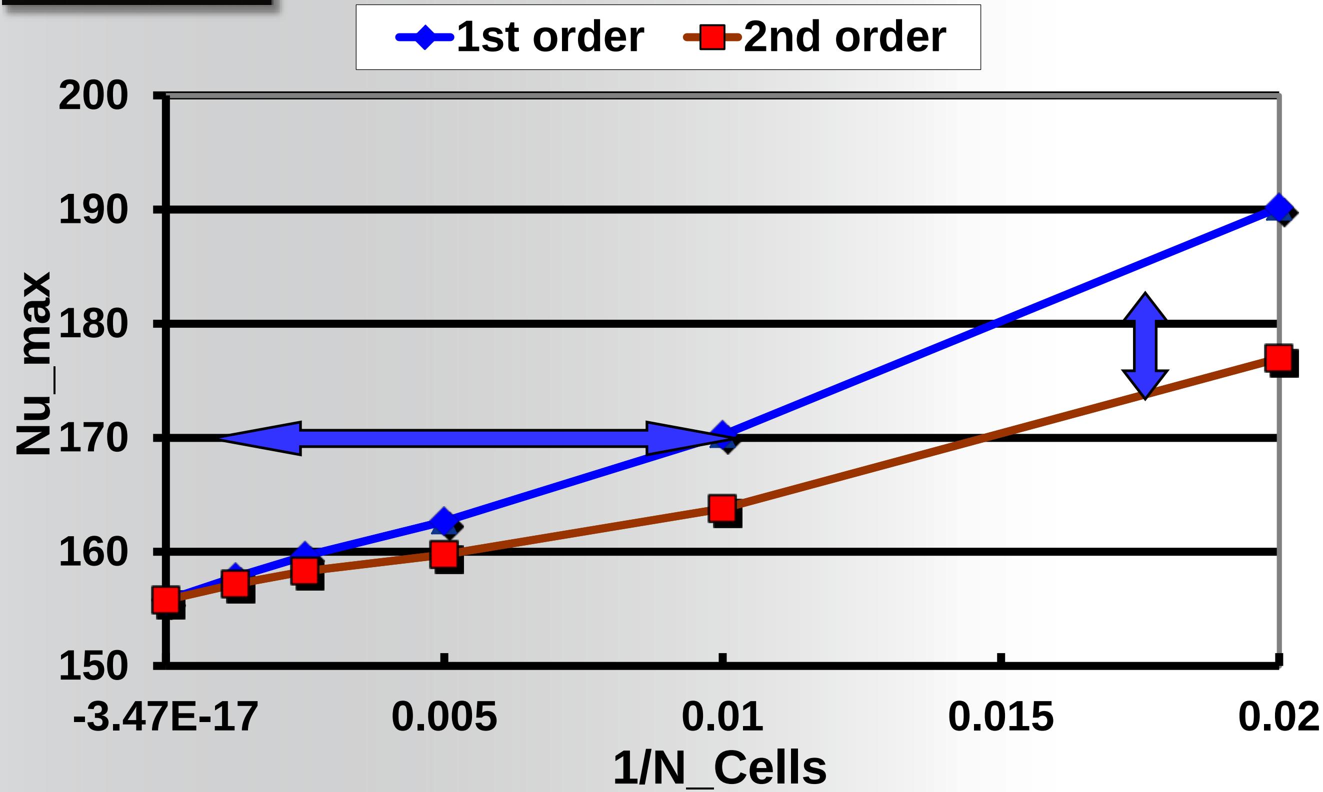
Discretization schemes:

- 1st order Upwind
- 2nd order Upwind



- Target quantities:
  - Heat transfer
  - Maximum Nusselt number

Discretization Error Estimation



The plot shows

- If the grid is fine enough, 1st and 2nd order solutions are the same
- On coarser meshes, the 2nd order solution is closer to the final solution

Practical alternatives for industrial cases are:

- Compare solutions from different order schemes
- Compare solutions on locally or regionally refined meshes

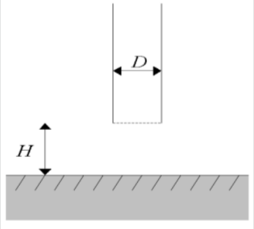
Model Errors

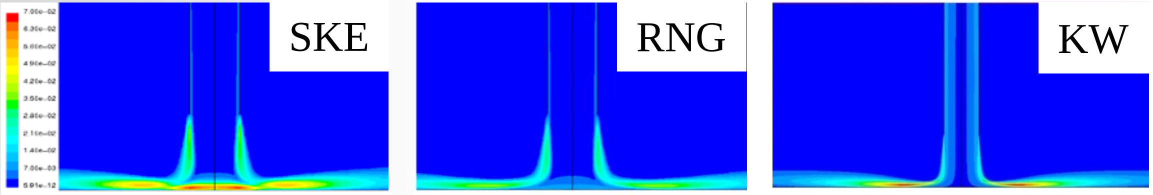
Inadequacies of (empirical) mathematical models:

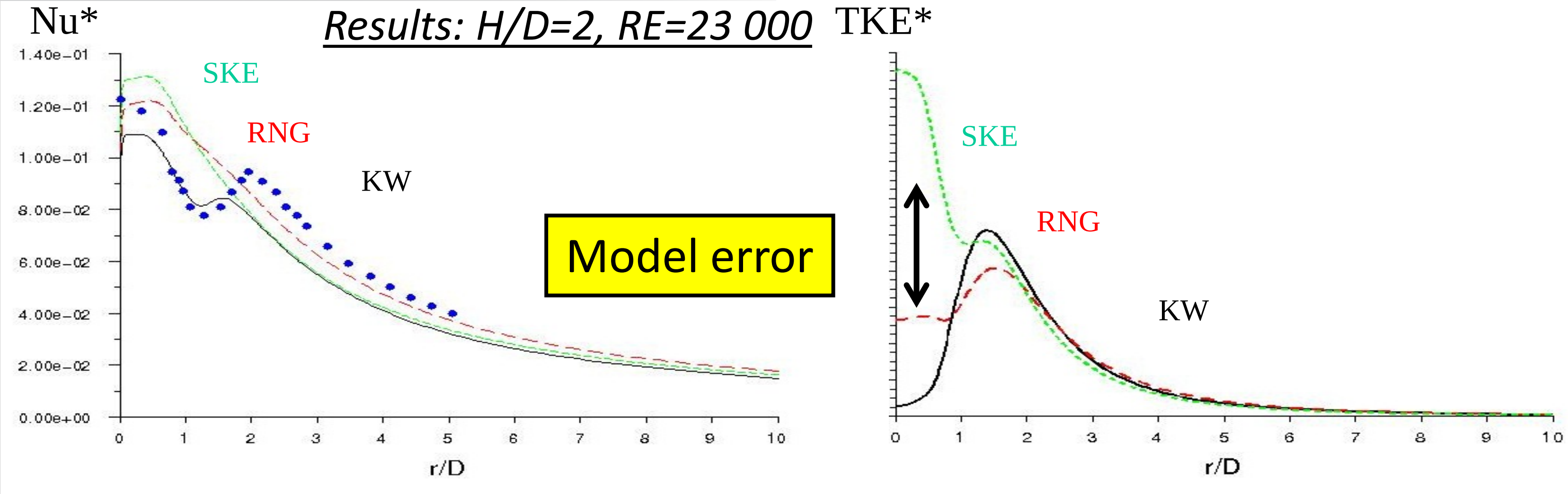
- Base equations (Euler vs. RANS, steady-state vs. unsteady-state, …)
- Turbulence models
- Combustion models
- Multiphase flow models
- …

Discrepancies between data and calculations remain, even after all numerical errors have become insignificant!

Model Error: Impinging Jet







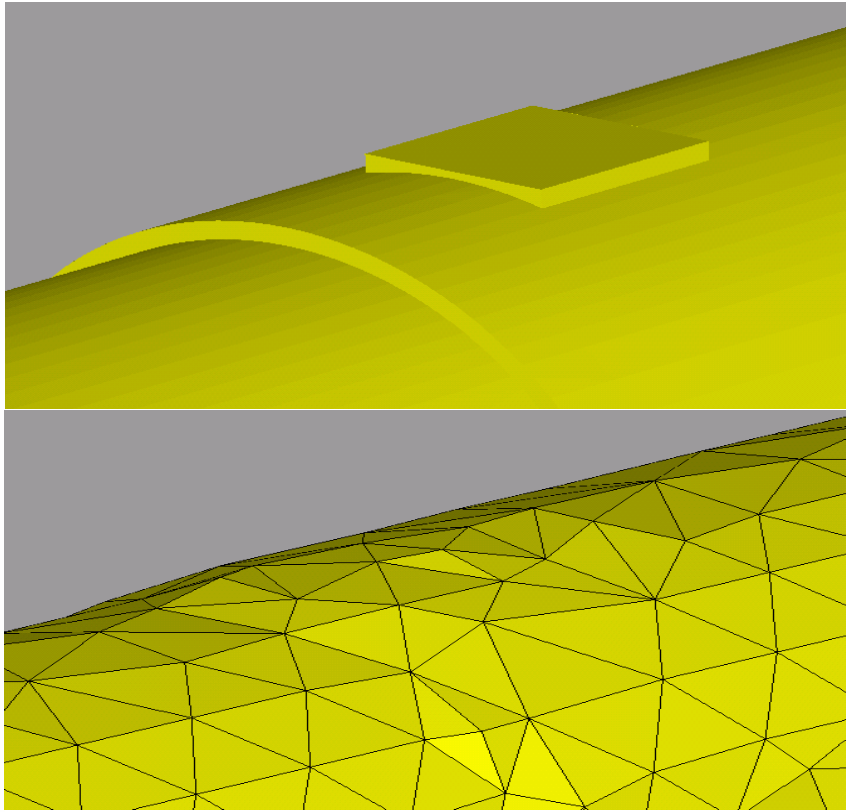
Systematic Errors

Discrepancies remain

- **even if numerical and model errors are insignificant**

'Systematic errors':

- **Approximations of:**
  - Geometry
  - Component vs. machine
  - Boundary conditions
  - Fluid and material properties, …



Try to 'understand' application and physics

Document and defend assumptions !

Perform <u>uncertainty analysis</u>

Meshing Best Practice Guidelines

Effects of low mesh quality:

- Discretization errors
- Round-off errors → Poor CFD results
- Convergence difficulties → Non-reliable CFD results
- Non-scalable meshes → Inconsistent CFD results on mesh refinement

Choose the appropriate meshing strategy

- Hex or Tet+Prism or Hybrid (use of non-conformal interfaces)
- Scalable grid quality (consistent grid quality on mesh refinement)

Meshing Best Practice Guidelines

Choosing your mesh strategy depends on

1.ACCURACY

Desired mesh quality
What is the maximum skewness and aspect ratio you can tolerate?

2. EFFICIENCY

Desired cell count
- Low cell count for resolving overall flow features vs High cell count for greater details

3. EASINESS TO GENERATE

Time available
- Faster Tet-dominant mesh vs crafted Hex/hybrid mesh with lower cell count

Goal: Find the best compromise between accuracy, efficiency and easiness to generate

Meshing: Capture Flow Physics

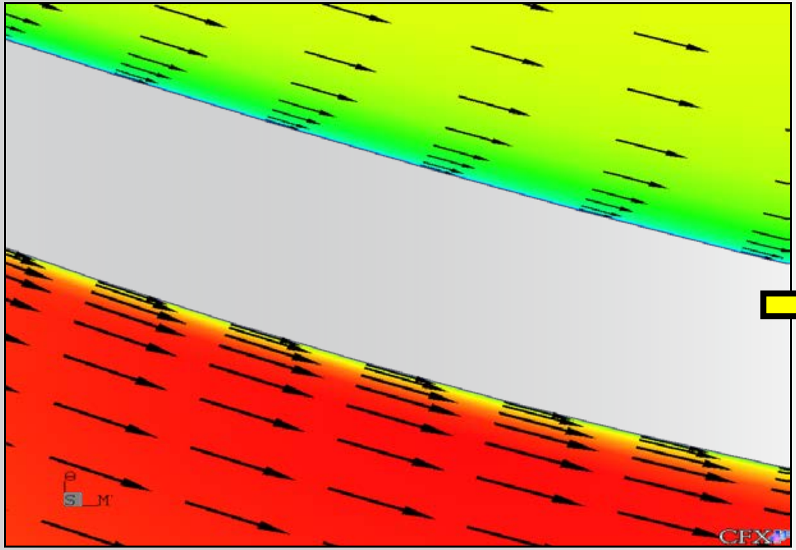
- **Grid must be able to capture important physics:**
  - Boundary layers
  - Heat transfer
  - Wakes, shock
  - Flow gradients

- **Recommended meshing guidelines for boundary layers**
  - Both the velocity and thermal boundary layers must be resolved
  - There should be a minimum of 10-15 elements across the boundary layer thickness
  - The mesh expansion ratio in the wall normal direction should be moderate:
    - $\leq$ 1.2 … 1.3
  - y+ $\approx$ 1 for heat transfer and transition modeling

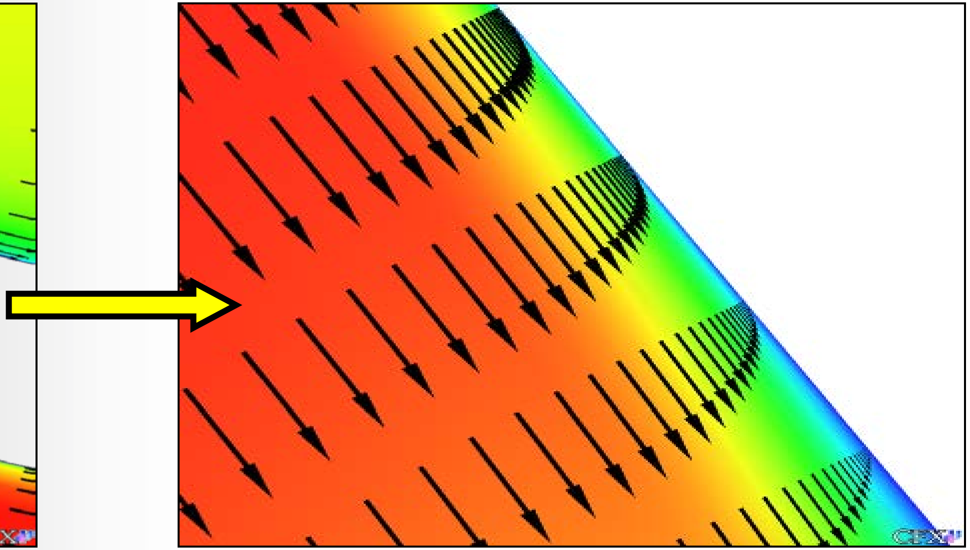
Meshing: Capture Flow Physics

- **Example: Velocity profiles at airfoil**

"Bad"



"Good"

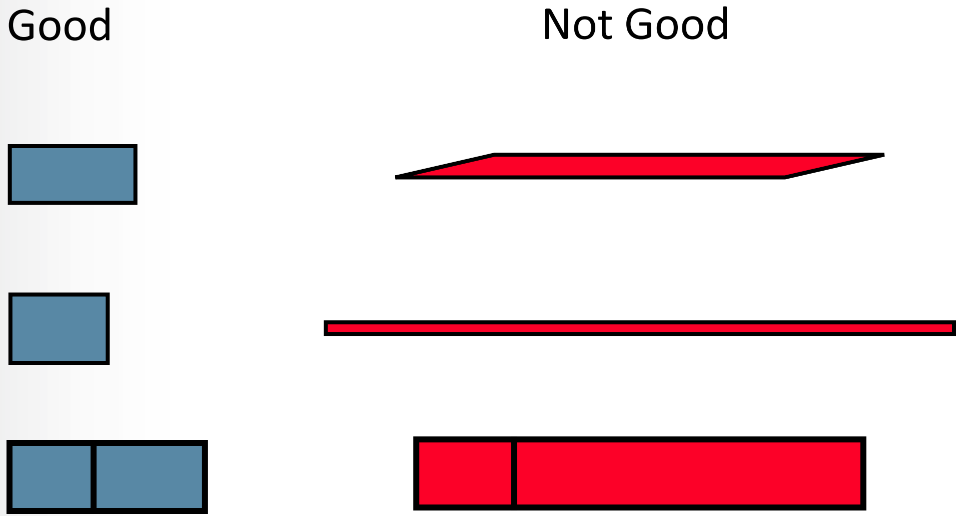


ANSYS

Mesh Quality

A good mesh depends on :

- **Cell not too distorted**
- **Cell not too stretched**
- **Smooth Cells transition**



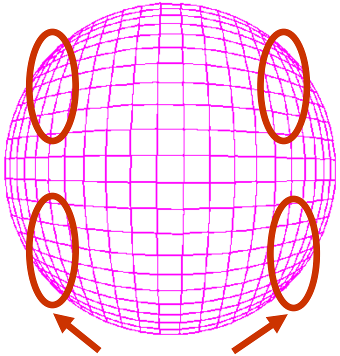
Mesh Quality

Grid generation:

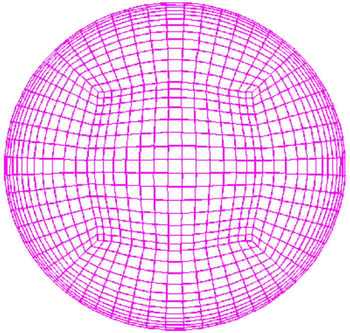
- Scalable grids
- Skewness < 0.95 (accuracy, convergence)
  - also worst Orthogonal Quality > .01 and average value much higher
- Aspect ratios < 100
- Expansion ratios < 1.5 …2
- Capture physics based on experience (shear layers, shocks)
- Angle between grid face & flow vector
- Concrete, quantitative recommendations for these factors presented in the Introduction to Ansys Meshing course are included in the appendix of this presentation

Grid refinement:

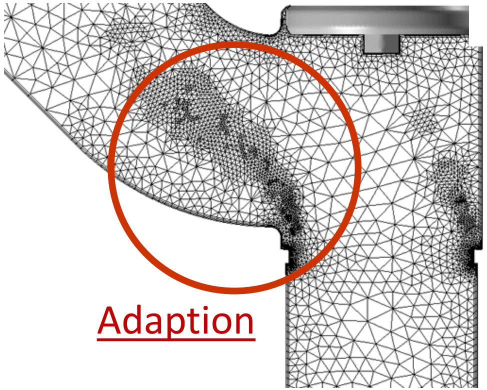
- Manual, based on error estimate
- Automatic adaptive based on ‘error sensor’



Bad cells



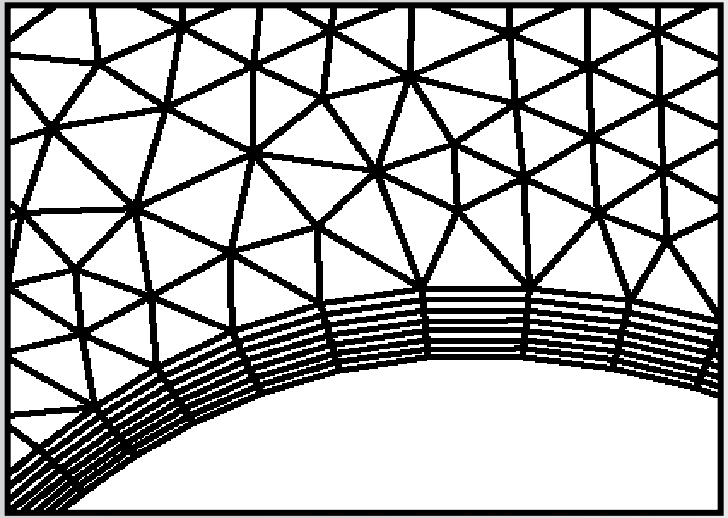
No Bad cells



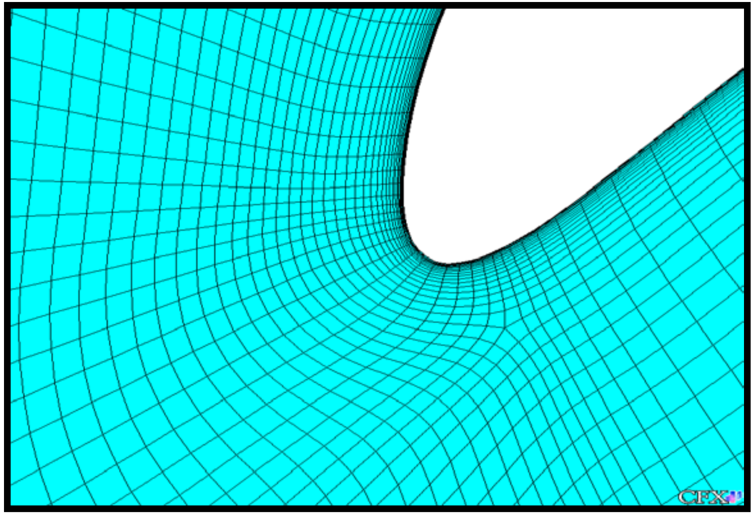
Adaption

Mesh Quality

Avoid sudden changes in mesh density



Not good



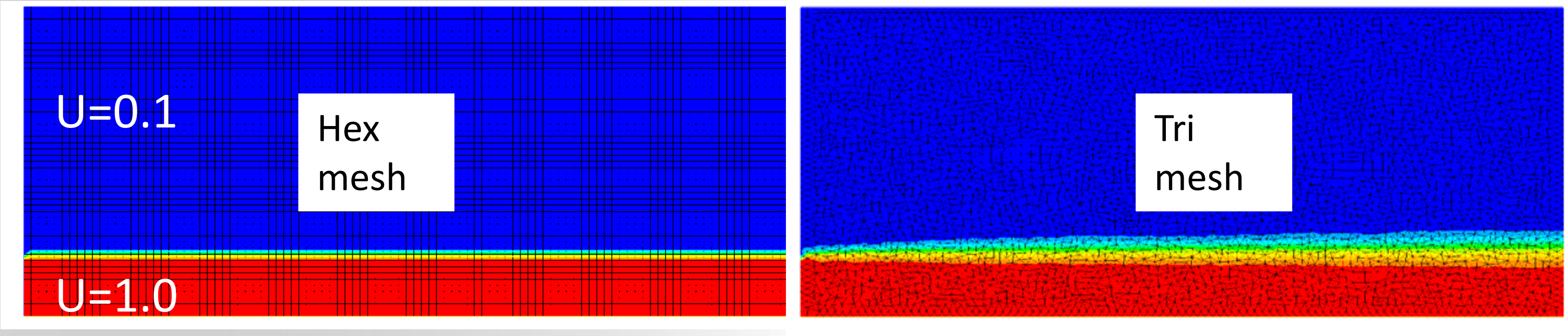
Good

Introduction | Error Types | **Best Practices for Meshing** | Summary

Hex vs Tet Mesh : Accuracy Comparison

- Direction of the flow well known

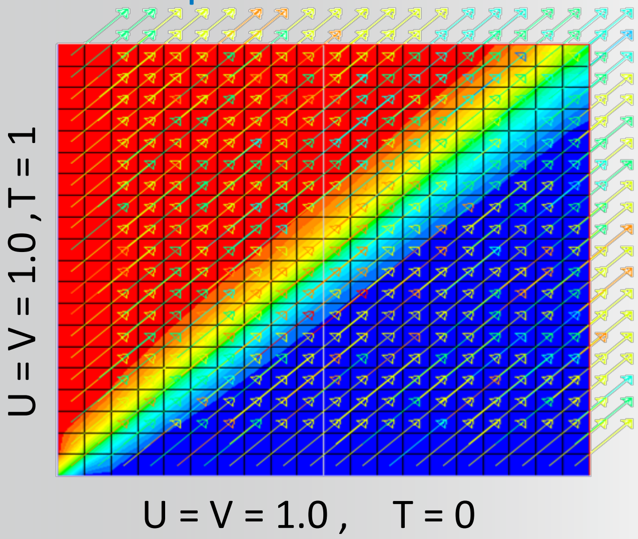
⇨ Quad/Hex aligned with the flow are more accurate than Tri with the same interval size

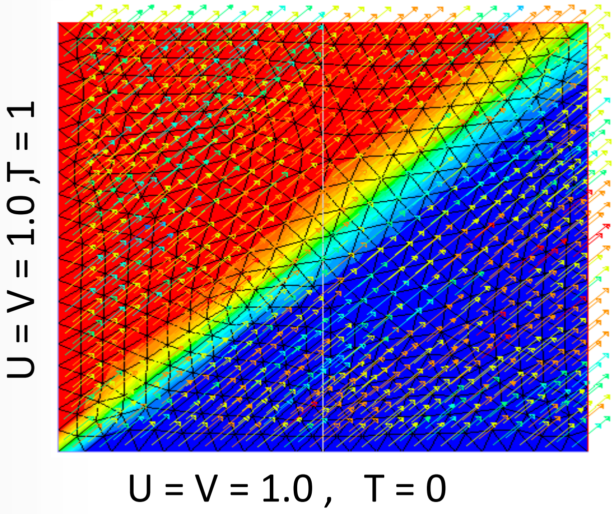


Contours of axial velocity magnitude for an inviscid co-flow jet

Hex vs Tet Mesh : Accuracy comparison

- For complex flows without dominant flow direction, Quad and Hex meshes lose their advantage
  - ⇨ Quad & Tri equivalent





Contours of temperature for inviscid flow

Summary

- **Try to 'understand' application and physics of the application**
- **Distinguish between numerical, model and other errors**
- **Document and defend assumptions**
  - Geometry
  - Boundary conditions
  - Flow regime (laminar, turbulent, steady-state, unsteady-state, …)
  - Model selection (turbulence, …)
- **Sources of systematic error**
  - Approximations
  - Data
- **Accuracy expectations vs. assumptions?**

Resources

ERCOFTAC SIG: ‚Quantification of Uncertainty in CFD‘

Roache, P.J., *Verification and Validation in Computational Science and Engineering*, Hermosa Publishers, 1998

ANSYS Best Practice Guidelines

Appendix

Impact of the Mesh Quality

Good quality mesh means that…

- Mesh quality criteria are within correct range
  - Orthogonal quality …
- Mesh is valid for studied physics
  - Boundary layer …
- Solution is grid independent
- Important geometric details are well captured

Bad quality mesh can cause;

- Convergence difficulties
- Bad physic description
- Diffuse solution

User must…

- Check quality criteria and improve grid if needed
- Think about model and solver settings before generating the grid
- Perform mesh parametric study, mesh adaption …

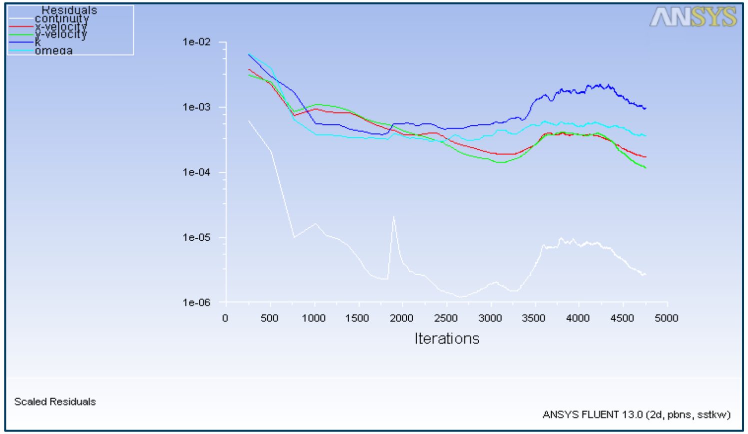
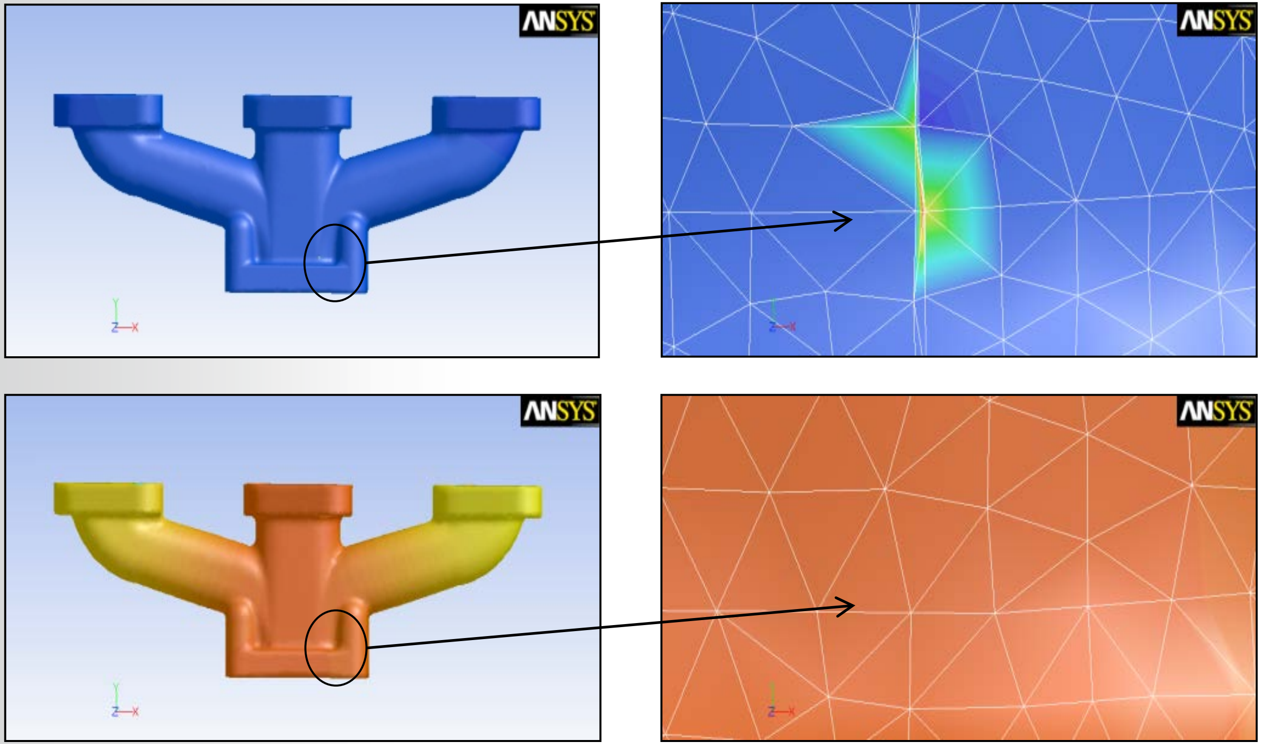


Table of Design Points

	A	B	C	D	E	F
1	Name	P1 - Sweep Method 3 Sweep Element Size	P2 - Sweep Method 2 Sweep Element Size	P3 - Sweep Method Sweep Element Size	P4 - Face Sizing Element Size	P6 - Dp
2		m	m	m	m	Pa
3	Current	0.04	0.04	0.04	0.02	747.88
4	DP 1	0.02	0.02	0.02	0.01	500.44
5	DP 2	0.01	0.01	0.01	0.005	361.4
6	DP 3	0.005	0.005	0.005	0.0025	307.6
7	DP 4	0.0025	0.0025	0.0025	0.00125	299.86
*						

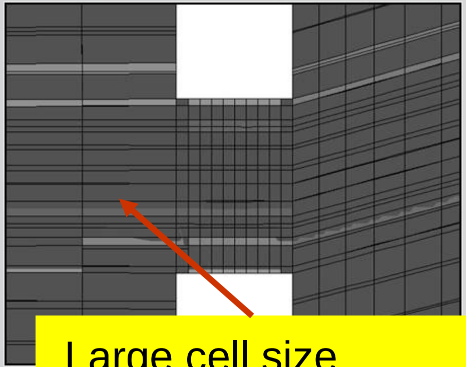
Impact of the Mesh Quality on the Solution

- **Example showing difference between a mesh with cells failing the quality criteria and a good mesh**
- **Unphysical values in vicinity of poor quality cells**

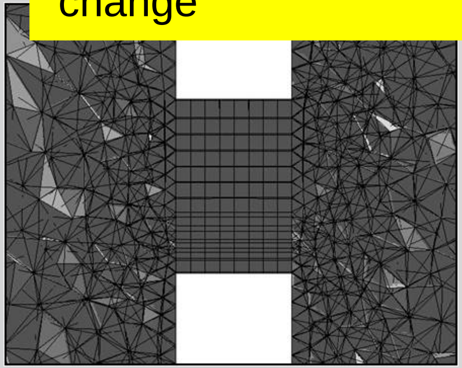


Impact of the Mesh Quality on the Solution

- Diffusion example

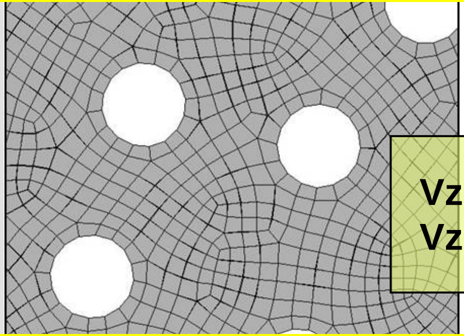


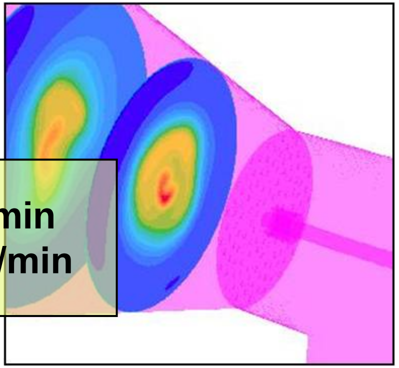
Large cell size change



Mesh 1

(max,avg)$_{CSKEW}$=(0.912,0.291)
(max,avg)$_{CAR}$=(62.731,7.402)

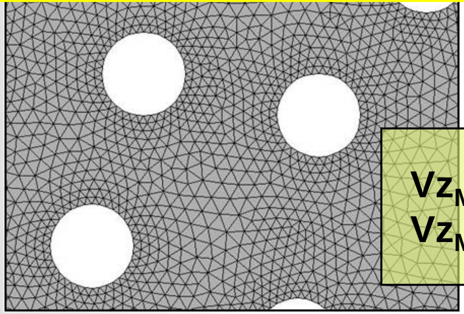


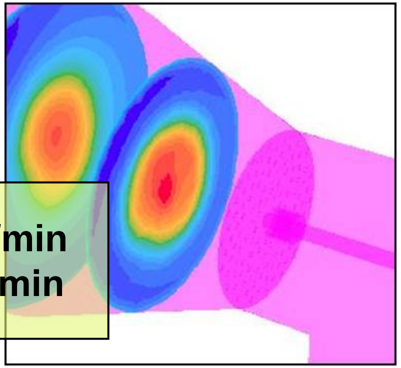


Vz_{MIN}≈-90ft/min
Vz_{MAX}≈600ft/min

Mesh 2

(max,avg)$_{CSKEW}$=(0.801,0.287)
(max,avg)$_{CAR}$=(8.153,1.298)





Vz_{MIN}≈-100ft/min
Vz_{MAX}≈400ft/min

Mesh Statistics and Mesh Metrics

Displays mesh information for Nodes and Elements

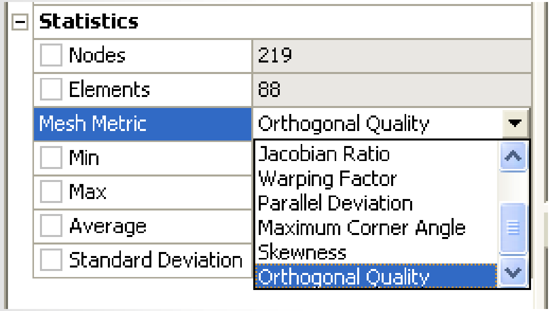
List of quality criteria for the Mesh Metric

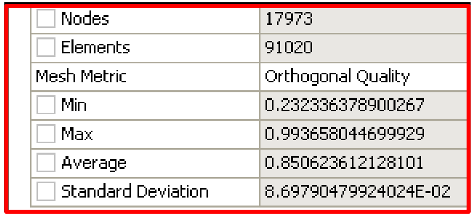
- **Select the required criteria to get details for quality**
- **It shows minimum, maximum, average and standard deviation**

Different physics and different solvers have different requirements for mesh quality

Mesh metrics available in ANSYS Meshing include:

- **Element Quality**
- **Aspect Ratio**
- **Jacobean Ration**
- **Warping Factor**
- **Parallel Deviation**
- **Maximum Corner Angle**
- **Skewness**
- **Orthogonal Quality**





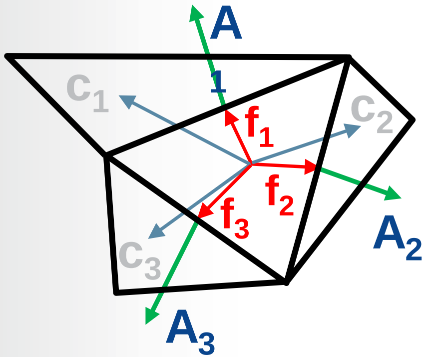
For Multi-Body Parts, go to corresponding body in Tree Outline to get its separate mesh statistics per part/body

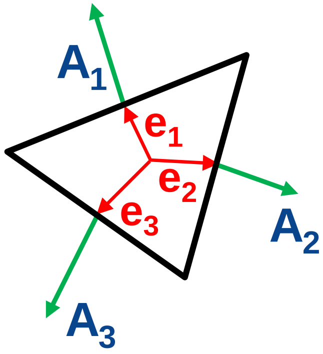
Mesh Quality Metrics

Orthogonal Quality (OQ)

On cell

On face





Derived directly from

Fluent solver discretization

- ***For a cell*** **it is the minimum of:**

$$\frac{A_i \cdot f_i}{|\vec{A_i}||\vec{f_i}|} \qquad \frac{A_i \cdot c_i}{|\vec{A_i}||\vec{c_i}|}$$

computed for each face *i*

For the face **it is computed as the minimum of** $\frac{A_i \cdot e_i}{|\vec{A_i}||\vec{e_i}|}$ **computed for each edge *I***

Where *Ai* is the face normal vector and *fi* is a vector from the centroid of the cell to the centroid of that face, and *ci* is a vector from the centroid of the cell to the centroid of the adjacent cell, where *ei* is the vector from the centroid of the face to the centroid of the edge

At boundaries and internal walls *ci* is ignored in the computations of OQ

Mesh Quality Metrics

Skewness

Two methods for determining skewness:

1. Equilateral Volume deviation:

$$\text{Skewness} = \frac{\text{optimal cell size} - \text{cell size}}{\text{optimal cell size}}$$

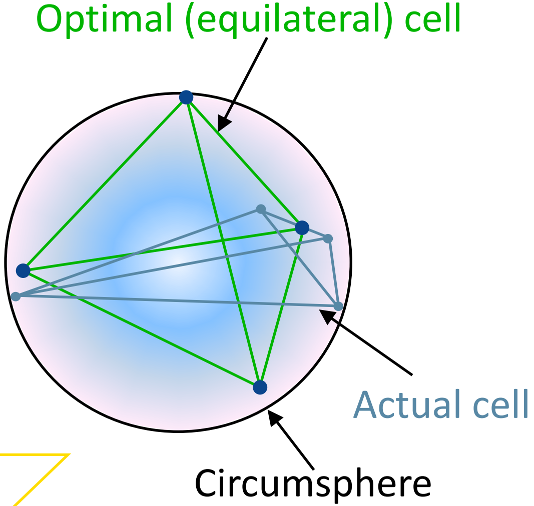
Applies only for triangles and tetrahedrons

2. Normalized Angle deviation:

$$\text{Skewness} = \max\left[\frac{\theta_{max} - \theta_e}{180 - \theta_e}, \frac{\theta_e - \theta_{min}}{\theta_e}\right]$$

Where θ_e is the equiangular face/cell (60 for tets and tris, and 90 for quads and hexas)

- Applies to all cell and face shapes
- Used for hexa, prisms and pyramids



Mesh Quality

Mesh quality recommendations

Low Orthogonal Quality or high skewness values are not recommended

Generally try to keep minimum orthogonal quality > 0.1, or maximum skewness < 0.95. However these values may be different depending on the physics and the location of the cell

Fluent reports negative cell volumes **if the mesh contains degenerate cells**

Skewness mesh metrics spectrum

Excellent	Very good	Good	Acceptable	Bad	Unacceptable
0-0.25	0.25-0.50	0.50-0.80	0.80-0.94	0.95-0.97	0.98-1.00

Orthogonal Quality mesh metrics spectrum

Unacceptable	Bad	Acceptable	Good	Very good	Excellent
0-0.001	0.001-0.14	0.15-0.20	0.20-0.69	0.70-0.95	0.95-1.00

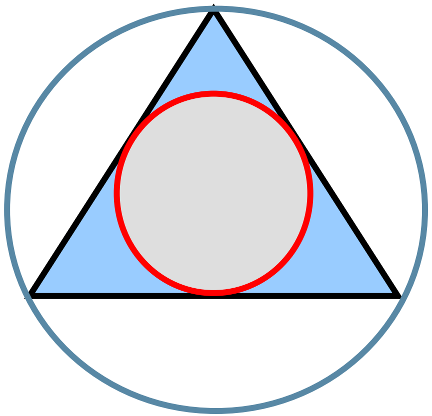
Aspect Ratio

2-D:

- Length / height ratio: $\delta x / \delta y$

3-D

- Area ratio
- Radius ratio of circumscribed / inscribed circle



Limitation for some iterative solvers

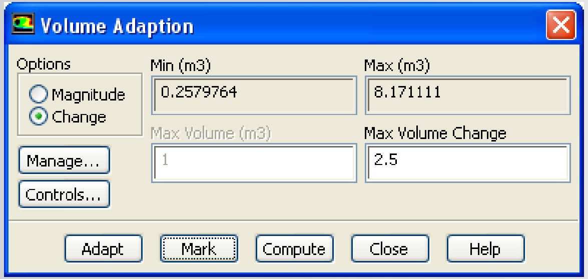
- A < 10 … 100
- (CFX: < 1000)

Large aspect ratio are accepted where there is no strong transverse gradient (boundary layer ...)

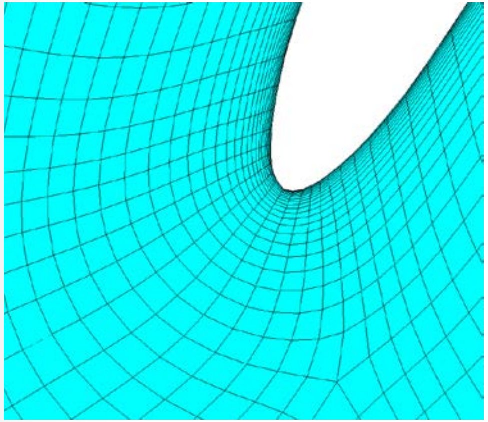
Smoothness

Checked in solver

- **Volume Change in Fluent**
  - Available in Adapt/Volume
  - *3D :* $\sigma_i = V_i / V_{nb}$

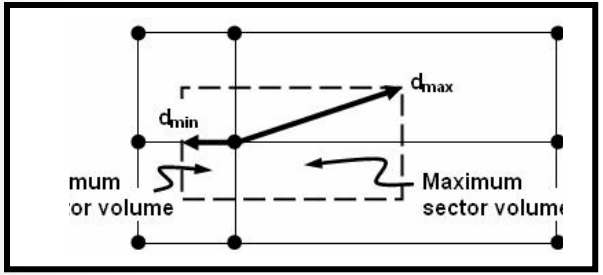


- **Expansion Factor in CFX**
  - Checked during mesh import
  - Ratio of largest to smallest element volumes surrounding a node



Recommendation:

Good: 1.0 < σ < 1.5
Fair: 1.5 < σ < 2.5
Poor: σ > 5 … 20



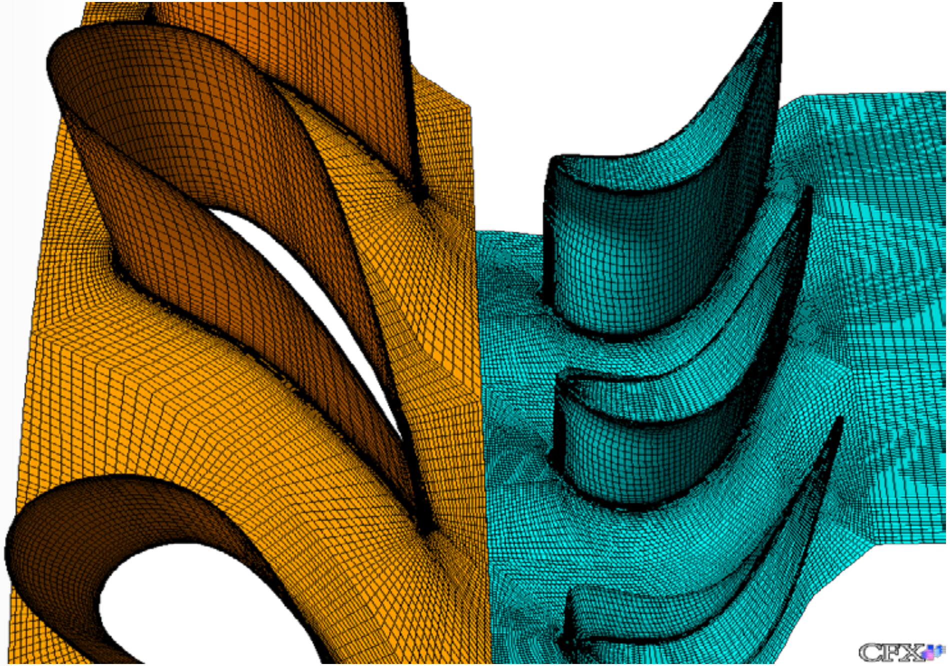
Elements: Hex

Pro:

- Good shear layer element
- Best element wrt. memory & calculation time per element

Con:

- Degree of automation for grid generation



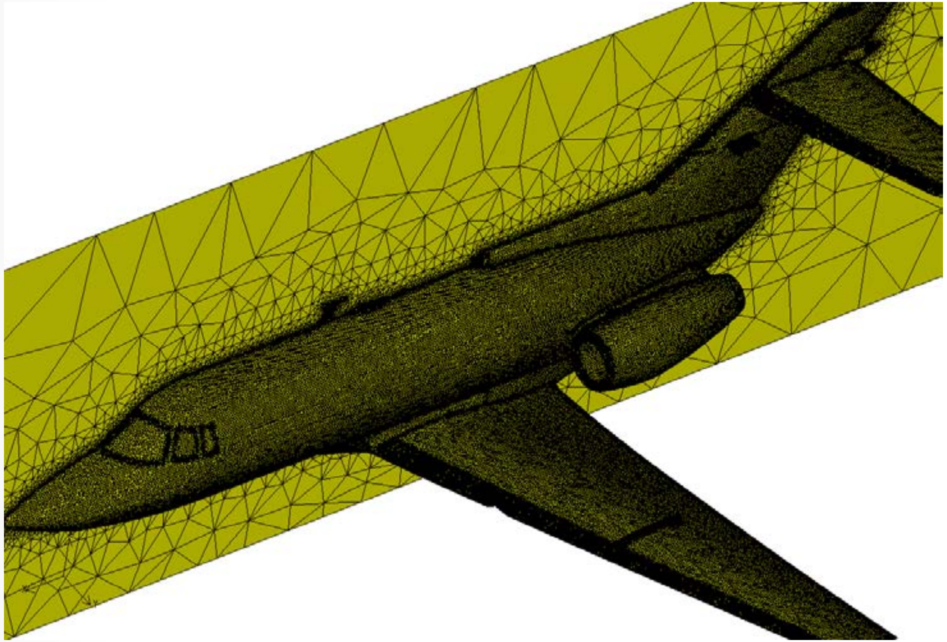
Elements: Tet

Pro:

- High degree of automation for grid generation

Con:

- Memory & calculation time per node ≈ 1.5 × hex
- Poor shear layer element
- No streamline orientation
- Quantity must (and can) make up for quality



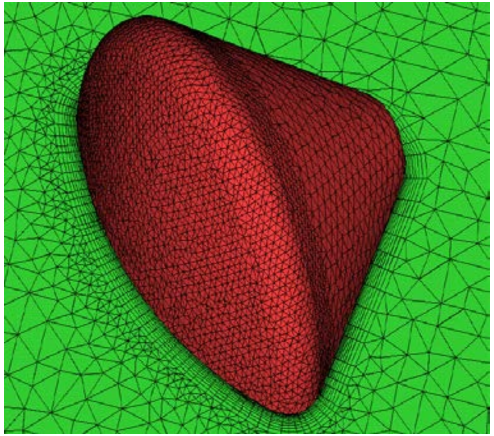
Elements: Prism

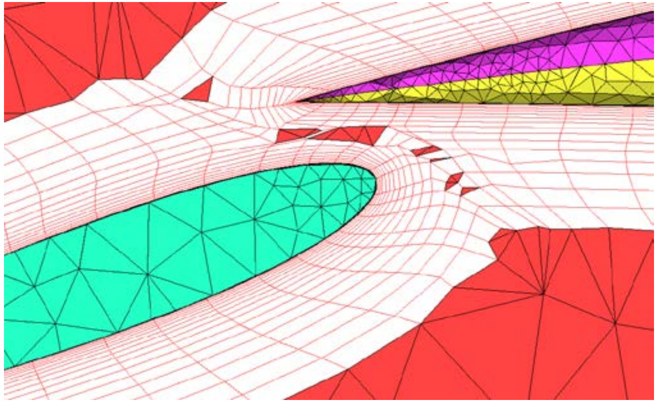
Pro:

- Better shear layer resolution than tet
- High degree of automation
- Tet/prism combination

Con:

- Less efficient than hex
- Topological difficulties (corners, …) → poor grid quality (angles, …)
- Manual repair





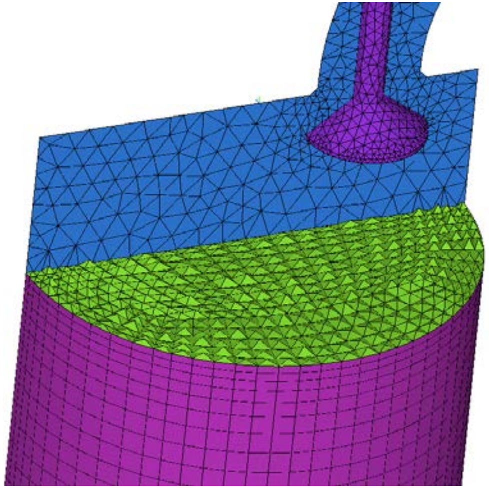
Elements: Pyramid

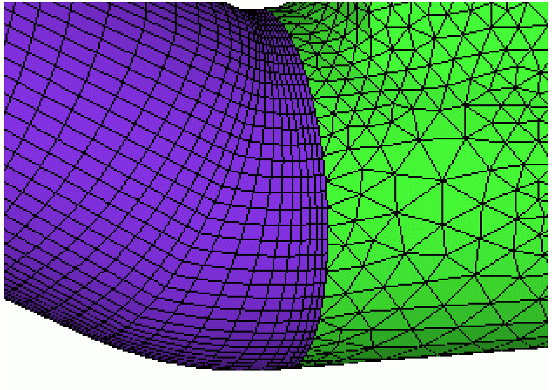
Use in hybrid grids

Transition element between hex and tet

Polyhedral grids

- ANSYS Fluent:
  - Generate base types
  - Convert
- ANSYS CFX builds polyhedrals around vertices





Recommendations

1st Option → Hex grid

- Best accuracy and numerical efficiency
- Time and effort manageable?

2nd Option → Tet/hex/pyramid grid

- Hex near walls & shear layers
- Developing technology …

3rd Option → Tet/prism grid

- High degree of automation
- Quality (prism/tet transition, …)

4th Option → Tet grid

- Shear layer resolution?

Grid Optimization

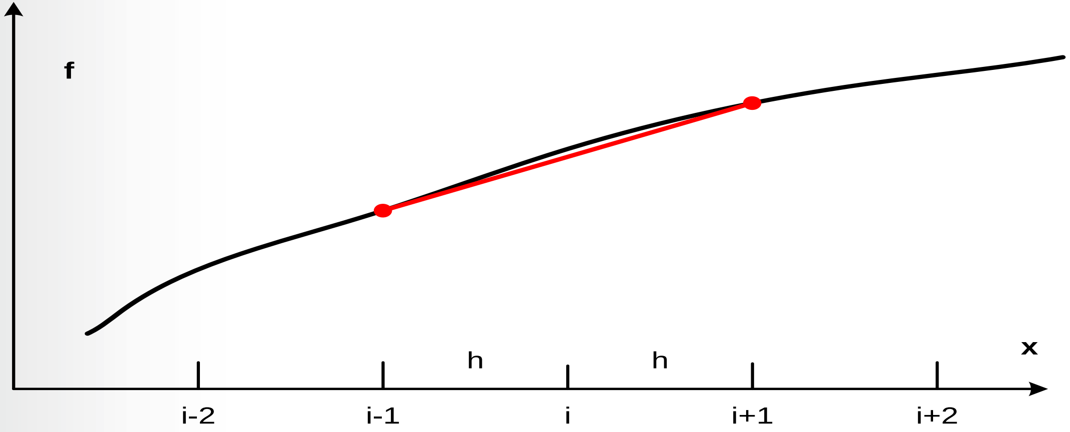
Truncation errors → source of discretisation errors

Minimize truncation errors → minimize discretisation errors

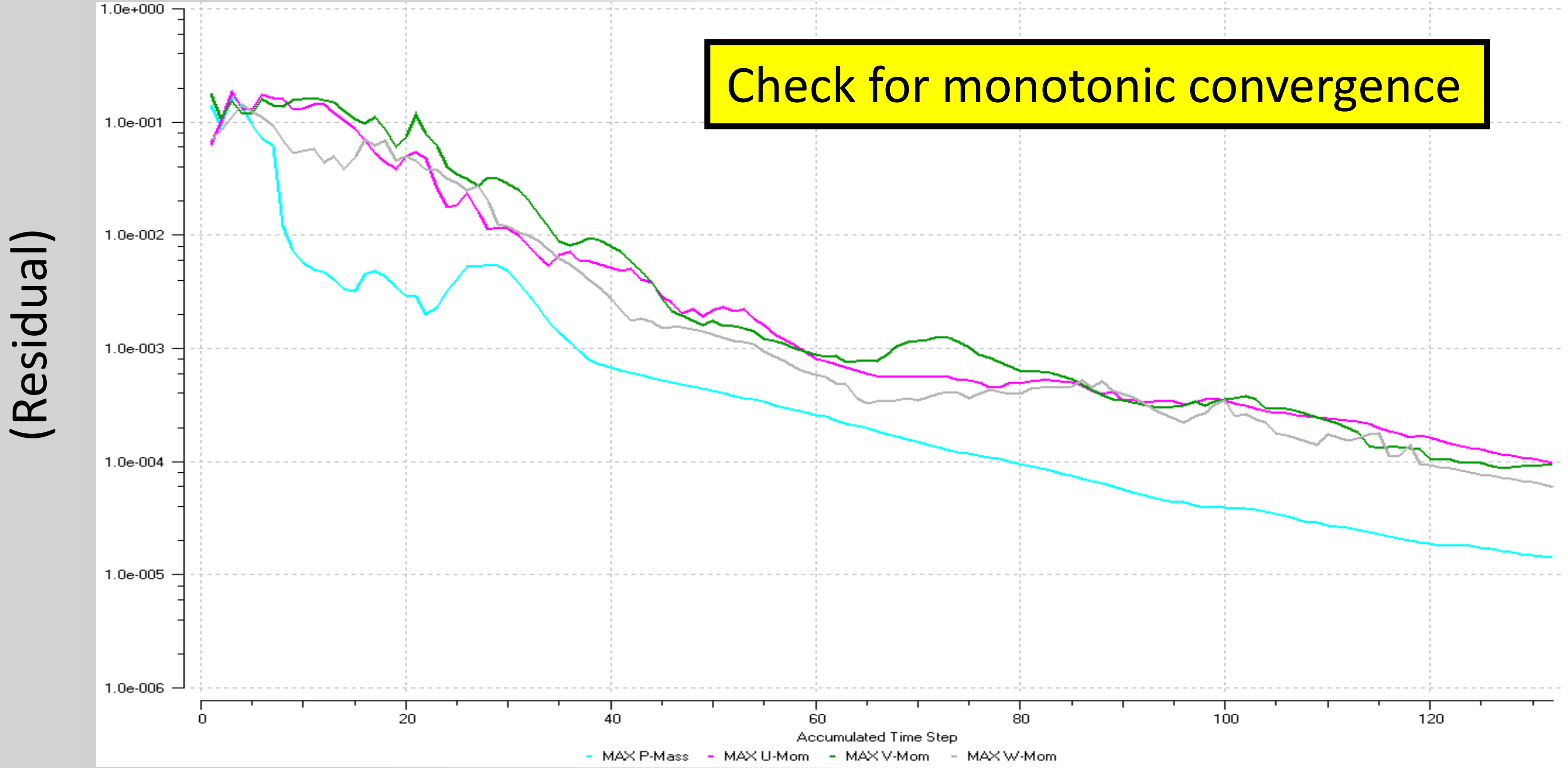
Truncation error → Difference between 'analog' and 'discrete' representation

$$\left(\frac{\partial f}{\partial x}\right)_i = \frac{f_{i+1} - f_{i-1}}{2h} + \tau_i$$

$$\tau_i = \frac{h^2}{6}\left(\frac{\partial^3 f}{\partial x^3}\right)_i + \ldots$$



Iteration Error – Example

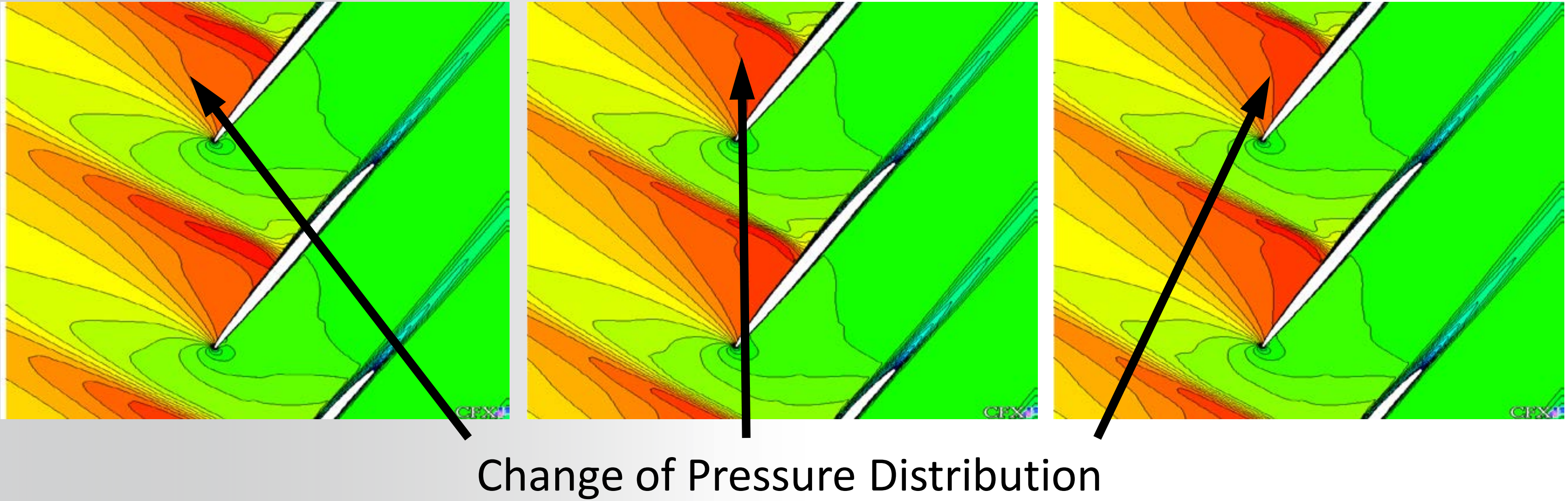


Iteration Error – Example

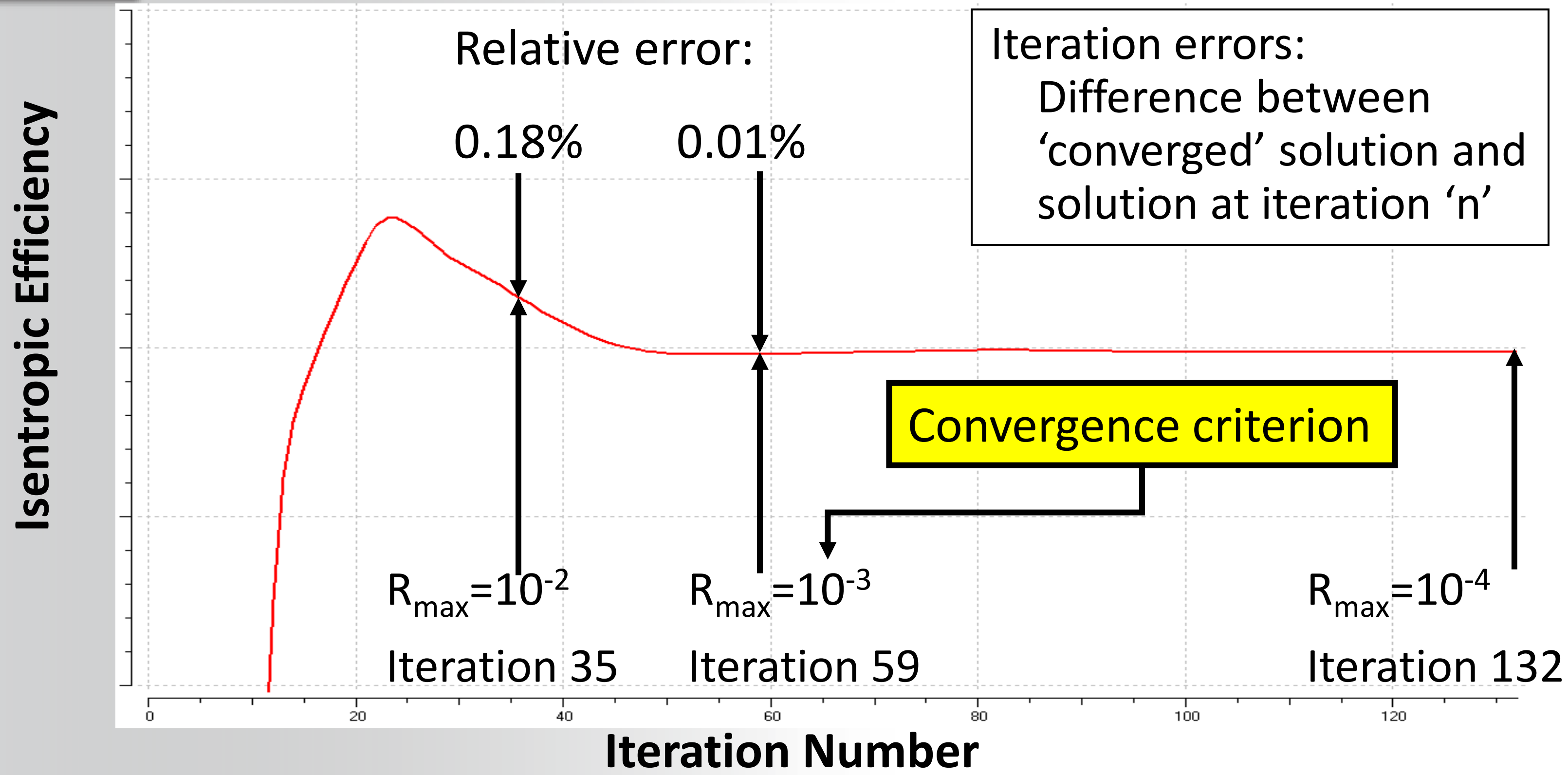
Effect of different residual limits during convergence:

- 2D Compressor cascade
- 2^{nd} order

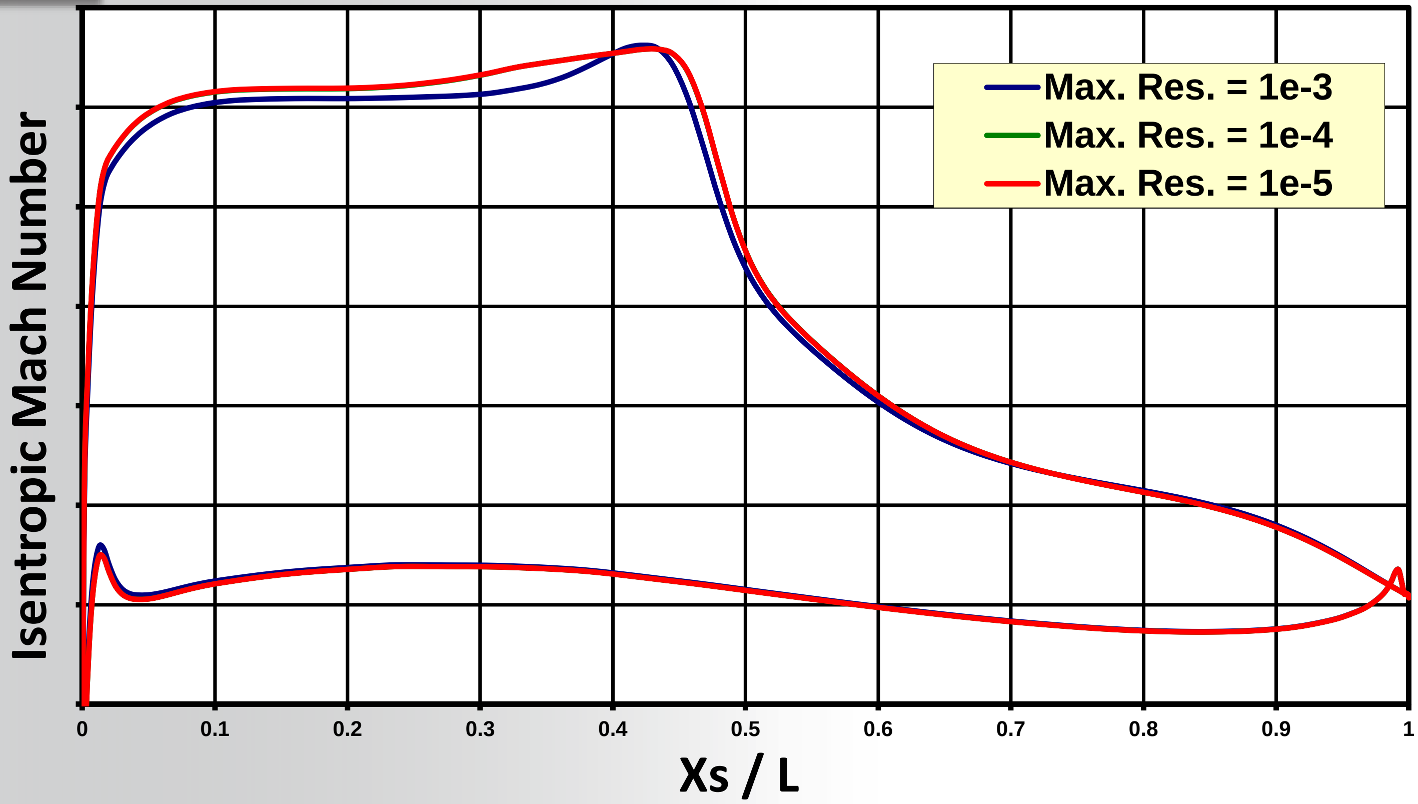
$R_{max} = 1 \times 10^{-3}$ $\qquad$ $R_{max} = 1 \times 10^{-4}$ $\qquad$ $R_{max} = 1 \times 10^{-5}$



Iteration Error – Example



Iteration Error – Example



Discretization Error Estimation

Grid	Nu		Error	
	1st order	2nd order	1st order	2nd order
50 × 50	190.175	176.981	22.1 %	13.6 %
100 × 100	170.230	163.793	9.3 %	5.1 %
200 × 200	162.664	159.761	4.4 %	2.6 %
400 × 400	159.646	158.296	2.3 %	1.4 %
800 × 800	157.808	157.168	1.1%	0.7 %
∞ × ∞	155.751	155.777		