Verification: Order of Convergence and Discretization Error

The order of grid (spatial) convergence involves the behavior of the discretization error term ε with regards to the grid spacing Δx,

$$\varepsilon = f(\Delta x) - f_{\text{exact}} = C \times (\Delta x)^p - \text{H.O.T.}$$

where p is the resulting order of convergence. Taking the logarithm, it becomes apparent how to find the value of p:

$$\log \varepsilon = \log C + p \log (\Delta x)$$

We simply need to find a few points of the curve $\log \varepsilon$ *v.* $\log (\Delta x)$ and take the slope to obtain p. If the grid refinement ratio r is constant, then the equation simply becomes:

$$p \log(r) = \log\left(\frac{f_3 - f_2}{f_2 - f_1}\right) .$$

We simply need to obtain the solutions f_i in order to calculate p. (Note that we can distinguish a *local* from a *global* order of accuracy by our choice of f_i—in general the global order of accuracy is one degree less than the local.)[1]

- We'll use a driven cavity flow for a square cavity. Load the geometry file for the 1 m×1 m square at `go.illinois.edu/me498cf-cavity`. The flow is laminar and the moving wall moves at the rate 0.0001 m/s for Re = 1000. (Set these conditions; use `air`.)

1. Solve the problem to convergence with mesh size 0.01 m, then refine the mesh to 2× resolution and solve the problem again. Now to 4× (original) resolution. Obtain the values f_i by querying the pressure at a selected point (which should be available as a cell vertex on all three grids, such as (0.25, 0.25)). Plot these using Excel or another tool to obtain p. Fill out the values below.
   a. point __________, __________, __________
   b. r = __________
   c. $f_{1\times}$ = __________
   d. $f_{2\times}$ = __________
   e. $f_{4\times}$ = __________
   f. p = __________
2. Does the problem converge at the same rate in other variables (such as velocity)? Elaborate.

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Validation: Comparison with Experimental Results

One chief challenge of validation of numerical experiments is the difference in reported quantities *v.* measurable quantities. Another challenge is that many CFD experiments are idealizations which cannot be physically reproduced. Sometimes this is sidestepped by validating with reference to a well-characterized benchmark problem, such as Rayleigh–Taylor mixing or 1D shock propagation. We will examine step 5 of the validation assessment process referenced in the lecture by comparing to experimental data on the driven square cavity problem.

Prasad & Koseff experimentally simulated a lid-driven cavity flow. Although they report a number of factors, the easiest ones for us to check are the x- and y-velocities along the respective centerlines. Their simulation is at much higher Re, so while we anticipate *qualitative* agreement our calculated values will not match theirs.

3. Plot the x- and y-velocities along the centerlines and compare the results to the plots on pp. 211–213 of Prasad & Koseff. (Submit your plots online.)

References

1. Slater, J. W. (cur.) (2008) Examining grid (spatial) convergence. NPARC Alliance CFD Verification and Validation Web Site. Available online at http://www.grc.nasa.gov/WWW/wind/valid/tutorial/spatconv.html.
2. CFD-Wiki. (2007) 2-D laminar/turbulent driven square cavity flow. Available online at http://www.cfd-online.com/Wiki/2-D_laminar/turbulent_driven_square_cavity_flow.
3. Prasad, A. K., Koseff, J. R. (1988) Reynolds number and end-wall effects on lid-driven cavity flow. *Physics of Fluids A 1*(2), pp. 208–218. Available online at https://www.researchgate.net/publication/238041375_Reynolds_number_and_end-wall_effects_on_a_lid-driven_cavity_flow.